

Grammar Hero's Reference Sheet

Slope of a Line

Let (x_1, y_1) and (x_2, y_2) be two points on the line.

$$\text{Slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: $ax^2 + bx + c = 0$ and $a \neq 0$

Equations of Lines

Standard Form: $Ax + By = C$

Slope-Intercept Form: $y = mx + b$
where $m = \text{slope}$ and $b = y - \text{intercept}$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$d = \text{distance between points } (x_1, y_1) \text{ and } (x_2, y_2).$

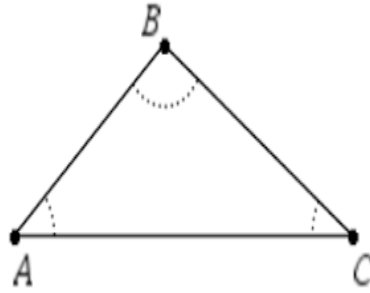
Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$M = \text{point halfway between points } (x_1, y_1) \text{ and } (x_2, y_2).$

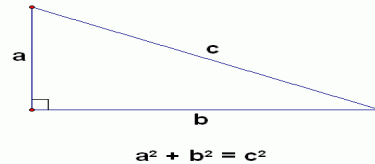
Triangle Sum Theorem

The sum of the three interior angles in a triangle is always 180° : $\angle a + \angle b + \angle c = 180^\circ$



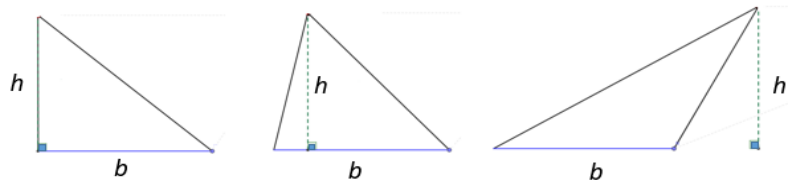
Pythagorean Theorem

In any right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse: $a^2 + b^2 = c^2$



Area of a Triangle

$$A = \frac{1}{2}bh$$



$b = \text{the base of any triangle}$
 $h = \text{perpendicular height of any triangle}$

Simple Interest Formula

$$I = prt$$

p = Principal (i.e., loan or investment amount)
I = Interest earned
r = Rate of interest per year in decimal form
t = Time in terms of years

Order of Operations (PEMDAS)

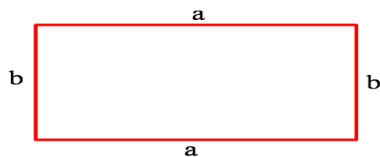
Order of operations refers to the order in which calculations are performed to evaluate an expression. The acronym PEMDAS is useful for remembering it.

P	Parenthesis
E	Exponents
M	Multiplication and Division (Left to Right)
D	Right)
A	Addition and Subtraction (Left to Right)
S	

Note: Multiplication and division have equal precedence, so their calculations are performed as they appear in the expression from left to right. Likewise, addition and subtraction have equal precedence, so their calculations are performed as they appear from left to right.

Perimeter

To find the perimeter of any polygon, excluding circles, you simply add up all of its sides. For example:



$$P = 2a + 2b = 2(a + b)$$

Note: The perimeter of a circle is its circumference.

Distance, Rate, and Time Formula

$$d = rt$$

d = distance,
r = rate
t = time

Rate and time must be in proportional units (e.g., if rate is given in terms of miles per hour, time must be in terms of hours)

Percent Change

$$PC = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100$$

PC = Percent Change
If PC is positive, there was an increase.
If PC is negative, there was a decrease

Statistics

$$\text{Mean} = \frac{\text{Sum of all Data Points}}{\text{Number of Data Points}}$$

Range = Maximum Value – Minimum Value

Mode = The value in the data set that occurs the most often

Median = The value in the middle of the data set

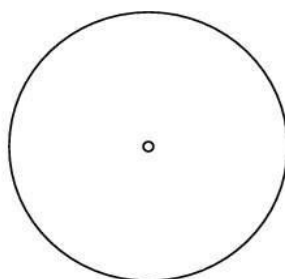
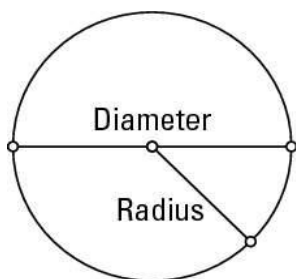
To find the median of a data set, arrange the observations in order from smallest to largest value. If there is an odd number of observations, the median is the middle value. If there is an even number of observations, the median is the average of the two middle values.

Circles

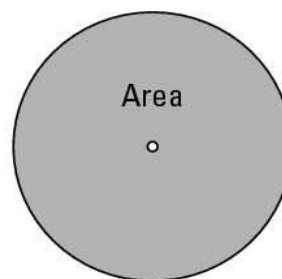
Area of a Circle:
 $A = \pi r^2$

Circumference of a Circle:
 $C = \pi d$ or $C = 2\pi r$

r = radius of a circle
 d = diameter of a circle
 $\pi = 3.14$ or $22/7$



Circumference



Area

Converting Units

Larger unit → smaller unit	<i>Multiply</i>
Smaller unit → Larger unit	<i>Divide</i>

Weight and Mass

1 Ton (T)	2,000 pounds
1 pound (lb)	16 ounces (oz)

Linear Units

12 inches (in)	1 foot (ft)
3 feet	1 yard (yd)
36 inches	1 yard
63,360 inches	1 mile (mi)
5,280 feet	1 mile
1,760 yards	1 mile

Time

1 day	24 hours
1 hour (hr)	60 minutes (min)
1 minute	60 seconds (sec)
1 year (yr)	365.25 days
1 week	7 days
1 year	12 months (mon)
1440 minutes	1 day
3600 seconds	1 hour

Capacity

8 fluid ounces	1 cup
2 cups	1 pint (pt)
2 pints	1 quart (qt)
4 quarts	1 gallon

Polygons

Shape	Number of Sides	Sum of Interior Angles
Triangle	3	180 degrees
Quadrilateral	4	360 degrees
Pentagon	5	540 degrees
Hexagon	6	720 degrees
Heptagon or Septagon	7	900 degrees
Octagon	8	1080 degrees
Any Polygon	n	$S = 180(n - 2)$

Multiplication Rules

The product and the quotient of one and any number is that number.

$$7 \times 1 = 7$$

$$100 \div 1 = 100$$

Zero times any number equals zero.

$$0 \times 2 = 0$$

$$858 \times 0 = 0$$

Zero divided by any nonzero number is zero.

$$0 \div 3 = 0$$

Dividing a number by zero is undefined.

$$\frac{5}{0} = \text{undefined}$$

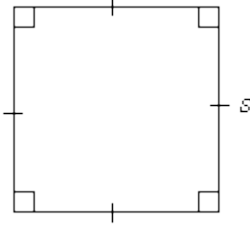
When multiplying or dividing with positives and negatives, use the signs charts.

Multiplication/Division Sign Chart

+	+	+
+	-	-
-	+	-
-	-	+

Quadrilaterals

Square

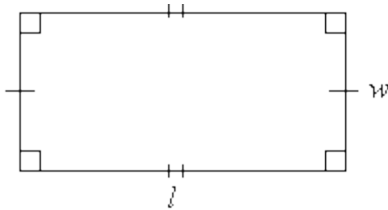


Perimeter: $P = 4s$

Area: $A = s^2$

Note: To find the perimeter of any quadrilateral, you simply add up all of its sides.

Rectangle

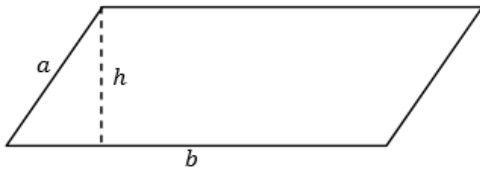


Perimeter: $P = 2l + 2w$

Area: $A = lw$

Note: To find the perimeter of any quadrilateral, you simply add up all of its sides.

Parallelogram

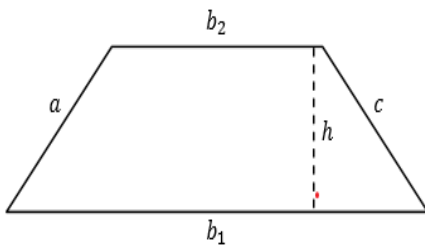


Perimeter: $P = 2a + 2b$

Area: $A = bh$

Note: To find the perimeter of any quadrilateral, you simply add up all of its sides.

Trapezoid



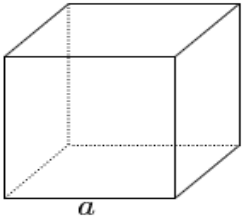
Perimeter: $P = a + b_1 + c + b_2$

Area: $A = \frac{1}{2} (b_1 + b_2) \cdot h$

Note: To find the perimeter of any quadrilateral, you simply add up all of its sides.

Formulas for Volume (V) and Surface Area (SA)

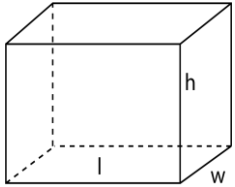
Cube



$$V = a^3$$

$$SA = 6a^2$$

Rectangular Solid



$$V = l \times w \times h$$

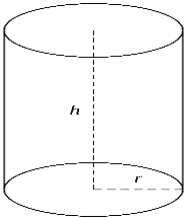
$$SA = 2(l \times w) + 2(w \times h) + 2(h \times l)$$

l = length

w = width

h = height

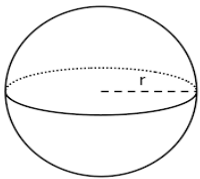
Cylinder



$$V = \pi r^2 h$$

$$SA = 2\pi r h + 2\pi r^2$$

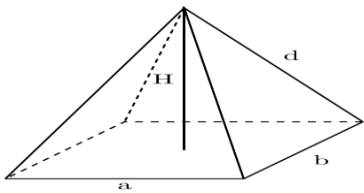
Sphere



$$V = \frac{4}{3} \pi r^3$$

$$SA = 4\pi r^2$$

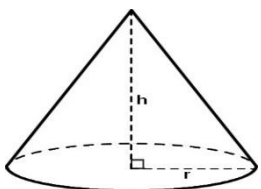
Rectangular Pyramid



$$V = \frac{1}{3} abh$$

Note: ab is the area of the base of the pyramid

Cone



$$V = \frac{1}{3} \pi r^2 h$$

Basic Probability

1. For any event A: $0 \leq P(A) \leq 1$
2. P(impossible event) = 0.
3. P(sure event) = 1.
4. $P(A) = \frac{\text{Desired outcome}}{\text{Total number of outcomes}}$
5. $P(\text{not } A) = 1 - P(A)$
6. $P(A \text{ and } B) = P(A) \times P(B)$ Independent (Replacement) vs. Dependent Events (No Replacement)
7. $P(A \text{ or } B) = P(A) + P(B)$ (Exclusive Events)
8. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (Non-Exclusive Events)

Prime Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Times Table

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Perfect Squares

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$

Laws of Exponents

<p>Zero-Exponent Rule</p> $a^0 = 1$ <p>Anything raised to the zero power is 1.</p>	$3^0 = 1$ $(5x^3y^4)^0 = 1$
<p>Power Rule</p> $(a^m)^n = a^{mn}$ <p>To raise a power to a power you need to multiply the exponents.</p>	$(x^5)^4 = x^{20}$ $(2x^4y^2)^3 = 2^3x^{12}y^6 = 8x^{12}y^6$ $\left(\frac{x^2}{y^5}\right)^4 = \frac{x^8}{y^{20}}$
<p>Negative Exponent Rule</p> $a^{-n} = \frac{1}{a^n}$ <p>Negative exponents in the numerator get moved to the denominator and become positive exponents. Negative exponents in the denominator get moved to the numerator and become positive exponents.</p>	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ $4x^{-2} = \frac{4}{x^2}$ $\frac{x^{-3}}{y^{-7}} = \frac{y^7}{x^3}$
<p>Product Rule</p> $a^m \cdot a^n = a^{m+n}$ <p>To multiply two exponents with the same base, you keep the base and add the powers.</p>	$x \cdot x^5 = x^6$ $y^4 \cdot y^9 = y^{13}$
<p>Quotient Rule</p> $\frac{a^m}{a^n} = a^{m-n}$ <p>To divide two exponents with the same base, you keep the base and subtract the powers.</p>	$\frac{x^5}{x^3} = x^2$ $\frac{y^4}{y^9} = \frac{1}{y^5}$ $\frac{x^3y^2}{x^2y^5} = \frac{x}{y^3}$

Writing Equations of Lines

Given the following, you can write equations of lines using these steps:

1. **The slope of the line and the y-intercept**

- a. Slope = m
- b. Y-intercept = b
- c. Plug the m and b values into $y = mx + b$

2. **The slope of the line and a point that lies on the line**

- a. Slope = m
- b. Substitute the point (x, y) in for x and y in the equation $y = mx + b$ and solve for b .
- c. Plug the m and b values into $y = mx + b$

OR

- a. Slope = m
- b. Substitute the slope and the point (x, y) in for x_1 and y_1 in the point-slope equation:

$$y - y_1 = m(x - x_1)$$
- c. Solve the equation for y .

3. **Two points that lie on the line**

- a. Using the slope formula: $m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$, find the slope.
- b. Substitute the slope and either point (x, y) in for x_1 and y_1 in the point-slope equation:

$$y - y_1 = m(x - x_1)$$
- c. Solve the equation for y .

Fractions

Adding and Subtracting Fractions	$\frac{a}{b} + \frac{c}{d} = \frac{(a \cdot d) + (c \cdot b)}{b \cdot d}$
Multiplying Fractions (Multiply straight across)	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$
Dividing Fractions (Keep, Change, Flip)	$\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$
Converting Mixed Numbers to Improper Fractions	$A\frac{b}{c} = \frac{(A \cdot c) + b}{c}$

Combined Rates & Work

Step 1: A problem involving combined rates or work can be solved using the formula:

$$\frac{T}{A} + \frac{T}{B} = 1$$

T = time working together

A = the time it takes for one person or thing to complete the task by themselves

B = the time it takes for one person or thing to complete the task by themselves

Step 2: Adjust the formula in accordance with the problem and solve

Example: Walter and Helen are asked to paint a house. Walter can paint the house by himself in 12 hours and Helen can paint the house by herself in 16 hours. How long would it take to paint the house if they worked together?

Step 1: A problem involving work can be solved using the formula:

$$\frac{T}{A} + \frac{T}{B} = 1$$

$$\frac{T}{12} + \frac{T}{16} = 1$$

Step 2: Solve the equation created in the first step. This can be done by first multiplying the entire problem by the common denominator and then solving the resulting equation. In this case, the least common denominator is 48.

$$48\left(\frac{T}{12} + \frac{T}{16} = 1\right)$$

$$4T + 3T = 48$$

$$7T = 48$$

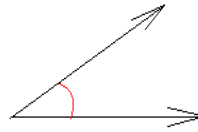
$$T = \frac{48}{7}$$

Step 3: Answer the question asked of you in the problem and be sure to include units with your answer.

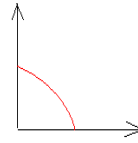
$$\text{Together} = \frac{48}{7} \approx 6.9 \text{ hours}$$

Angle Types & Special Angle Pairs

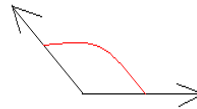
Acute Angle: An angle whose measure is less than 90 degrees.



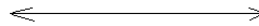
Right angle: An angle whose measure is 90 degrees.



Obtuse angle: An angle whose measure is bigger than 90 degrees but less than 180 degrees.

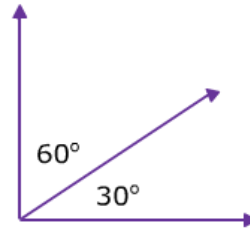


Straight angle: An angle whose measure is 180 degrees. Thus, a straight angle look like a straight line.



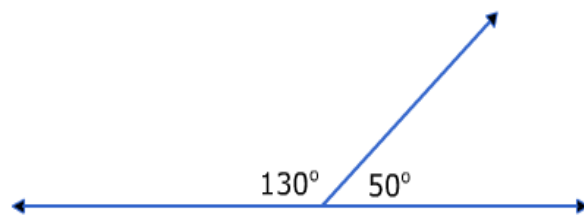
Complementary angles: Two angles that have a sum of 90 degrees.

$$\angle 1 + \angle 2 = 90^\circ$$



Supplementary angles: Two angles that have a sum of 180 degrees.

$$\angle 1 + \angle 2 = 180^\circ$$



Properties of Radicals

A. $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example: $x^{\frac{1}{3}} = \sqrt[3]{x}$

B. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Example: $x^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$

C. $\sqrt[n]{a^n} = a^{\frac{n}{n}} = a^1 = a$

Examples: $\sqrt{x^2} = x$ $\sqrt[5]{x^5} = x$

D. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Examples:

1. $\sqrt{36y^4} = \sqrt{36} \cdot \sqrt{y^4} = 6y^2$

4. $\sqrt[4]{64x^5y^8} = \sqrt[4]{16x^4y^8} \cdot \sqrt[4]{4x} = 2xy^2\sqrt[4]{4x}$

2. $\sqrt{72y^5} = \sqrt{36y^4} \cdot \sqrt{2y} = 6y^2\sqrt{2y}$

5. $\sqrt[5]{64x^5y^8} = \sqrt[5]{32x^5y^5} \cdot \sqrt[5]{2y^3} = 2xy^5\sqrt[5]{2y^3}$

3. $\sqrt[3]{48y^7} = \sqrt[3]{8y^6} \cdot \sqrt[3]{6y} = 2y^2\sqrt[3]{6y}$

E. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad b \neq 0$

Examples:

1. $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

2. $\sqrt{\frac{x^2}{4y^2}} = \frac{\sqrt{x^2}}{\sqrt{4y^2}} = \frac{x}{2y}$

3. $\sqrt[3]{\frac{8y^4}{27x^3}} = \frac{\sqrt[3]{8y^4}}{\sqrt[3]{27x^3}} = \frac{\sqrt[3]{8y^3 \cdot y}}{\sqrt[3]{27x^3}} = \frac{2y\sqrt[3]{y}}{3x}$

FACTORIZING GUIDE

- I. Check for GCF**
II. Count the number of terms

Two Terms

1. Difference of 2 Squares

$$A^2 - B^2 = (A - B)(A + B)$$

* Note: $A^2 + B^2$ is prime and does **NOT** factor

Example:

Since $A = 2x$ and $B = 5y$

$$4x^2 - 25y^2 = (2x - 5y)(2x + 5y)$$

2. Sum or Difference of Cubes

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

Examples:

$$x^3 - 8 \text{ here } A = x \text{ and } B = 2$$

$$\text{So } x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

$$27y^3 + 64z^3 \text{ here } A = 3y \text{ and } B = 4z \text{ so}$$

$$27y^3 + 64z^3 = (3y + 4z)(9y^2 - 12yz + 16z^2)$$

Three Terms

1. Leading Coefficient is 1

a) $x^2 + x - 42 = (x - 6)(x + 7)$

(find 2 numbers that multiply out to -42 and add up to 1)

b) $x^2 - 9x + 14 = (x - 7)(x - 2)$

(find 2 numbers that multiply to 14 and combine to -9)

2. Leading Coefficient is NOT 1

a) **Check to see if it is a perfect square trinomial**

(use the sum or difference of the square root of the first and last terms)

$$4x^2 - 20xy + 25y^2 = (2x - 5y)(2x - 5y) = (2x - 5y)^2$$

b) **Use trial and error to factor the form $Ax^2 + Bx + C$**

(If A and C are small or prime numbers, try different combinations to get the outer and inner terms to equal B.)

$$2x^2 - 5x - 7 = (2x - 7)(x + 1) \text{ not } (2x + 1)(x - 7) \\ \text{not } (2x - 1)(x + 7)$$

(outers = 2x, inners = -7x, their sum is -5x)

*Note: If $B^2 - 4AC$ is not a perfect square, the trinomial is prime.

c) **Use the AC (Australian) Method**

1. Find the product of A times C and list pairs of factors

Select the pair of factors whose sum is B.

(If none match, then the trinomial is prime and cannot be factored.)

2. Rewrite the trinomial as four terms with the pair of factors as coefficients of the middle term.

$$4x^2 - 13x + 10 \quad AC = 40 \quad B = -13$$

-2	-20	sum to -22
-4	-10	sum to -14
-5	-8	sum to -13

use this pair →

3. See factoring four terms **Two-by-Two** Ex.1 below.

Four Terms **Factor by Grouping**

Examples:

1. Two by Two:

1) $4x^2 - 8x - 5x + 10 = 4x(x - 2) - 5(x - 2) = (x - 2)(4x - 5)$

2) $x^3 - 3x^2 + 5x - 15 = x^2(x - 3) + 5(x - 3) = (x - 3)(x^2 + 5)$

2. Three by One: $x^2 - 6x + 9 - y^2 = (x - 3)^2 - y^2 = (x - 3 - y)(x - 3 + y)$